Design of three-mirror telescopes via a differential equation method

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ABSTRACT

A differential equation method is applied to the design of a three-mirror telescope. The resulting system is mostly free of spherical aberration, coma and astigmatism. From caustic theory and a generalization of the Coddington Equations, the Abbe sine condition and the constant optical path length condition, three coupled differential equations, one for each reflecting surface, are generated. A system which satisfies these conditions will have a high resolution over a wide field of view. Analysis of this application is presented as a comparison to a similar three-mirror telescope system produced by conventional optimization techniques.

KEYWORDS: telescope, coddington, reflecting, mirror, wide field, aplanatic, abbe sine, anastigmatic, differential equations

1. INTRODUCTION

The Coddington Equations[1] are used to calculate the astigmatism of a purely spherical system where astigmatism is taken to mean the off-axis aberration measured by the separation between the meridional and sagittal foci of the system.[2] Starting with caustic theory, Burkhard and Shealy[3] produced a means of describing the principal curvatures of a wavefront as it propagates through an *aspherical* optical system. These are known collectively as the *Generalized Coddington Equations* (GCEs). GCEs provide a powerful means of calculating a local wavefront's principal radii of curvature and thus the various aberrations for a wide variety of systems; most importantly, GCEs can be used as a condition to eliminate aberrations from a system altogether.[5,6]

Korsch[4,7] designs a three-mirror telescope by applying the conditions of aplanatism (constant optical path length and the Abbe sine condition) to the second and third mirrors. The shape of the primary mirror is optimized to obtain the desired system performance. While Korsch's method can be used to produce high-performance systems, one wonders whether similar superior systems can be produced by purely analytical means. We show that the aplanatism condition, coupled with the GCEs, can produce a set of unique differential equations which

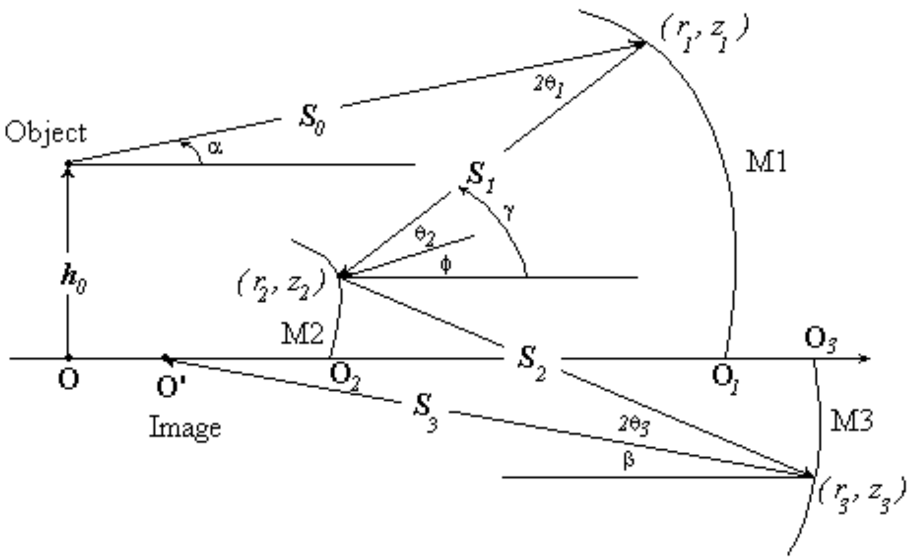

Figure 1: The configuration of a three-mirror microscope.

can be solved to produce a system free of spherical aberration, coma and astigmatism without having to resort to optimization.

2. THEORY

To design a three-mirror telescope which is free of spherical aberration, one must satisfy the constant optical path length condition[4,7]

$$\left|\vec{S_0}\right| + \left|\vec{S_1}\right| + \left|\vec{S_2}\right| + \left|\vec{S_3}\right| = c = |d_0| + |d_1| + |d_2| + |d_3|, \qquad (1)$$

where

$$d_0 = \overline{OO_1},\ d_1 = -\overline{O_1O_2},\ d_2 = \overline{O_2O_3},\ d_3 = \overline{O_3O'}.$$

To eliminate coma, the Abbe sine condition[4,7],

$$r_1 = f\ sin\beta, \qquad (2)$$

must be maintained. Here, r_1 and β are as in Fig. 1, and f is the focal length of the system.

The Generalized Coddington Equations[3] at the third reflecting surface are given by

$$\frac{1}{r'_T} = \frac{1}{r_T} + \frac{2}{R_T cos\theta_3} \tag{3}$$

$$\frac{1}{r'_S} = \frac{1}{r_S} + \frac{2cos\theta_3}{R_S} \tag{4}$$

where r_T and r_S are the meridional and sagittal radii of curvature of the incident wavefront, respectively. r'_T and r'_S are the meridional and sagittal radii of curvature of the reflected wavefront. θ_3 is the reflecting angle as shown in Fig 1. The principal radii of curvature of the reflecting surface, R_T for the meridional plane and R_S for sagittal plane, are given by

$$R_S = \frac{\rho\sqrt{1 + z'^2_3}}{z'_3} \tag{5}$$

$$R_T = \frac{(1 + z'^2_3)^{3/2}}{z''_3}, \tag{6}$$

where the reflecting surface is described by the formula $z = f(p)$. z is the surface sag and p is the radial distance measured from any point on the surface to the optical axis.

If both the meridional and sagittal rays focus at the same point, then the reflecting wavefront will be free of astigmatism. Setting $r'_T = r'_S$ in Eqs. 3 and 4 gives[5,6]

$$\frac{2cos\theta}{R_S} - \frac{2}{R_T cos\theta} = \frac{1}{r_T} - \frac{1}{r_S} = w. \tag{7}$$

The value of w is a measure of the astigmatism of the incident wavefront, which can be related to the wavefront of rays incident at the center of the entrance pupil by the following:

$$w = \frac{1}{r_{T0} + l} - \frac{1}{r_{S0} + l}, \tag{8}$$

where r_{T0} and r_{S0} are the meridional and sagittal radii of curvature of the wavefront at the center of the entrance pupil, respectively. l is the distance between the center of the entrance pupil and the reflecting point on the surface.

By combining Eqs. 5, 6 and 7, one obtains a second-order differential equation for the third reflecting surface:[5,6]

$$z''_3 = \frac{z'_3(1 + z'^2_3)cos^2\theta_3}{r_3} - \frac{\omega}{2}(1 + z'^2_3)^{3/2}cos\theta_3. \tag{9}$$

By setting $w = 0$, one can eliminate astigmatism from the system.

In general, one obtains three coupled differential functions for each reflecting surface by applying the various boundary conditions necessary to produce a system with the desired performance:

$$F(z_1, z_1', z_2, z_2', r_1, r_2) = 0 \tag{10}$$

$$G(z_2, z_2', z_3, z_3', r_2, r_3) = 0 \tag{11}$$

$$H(z_3, z_3', z_1, z_1', r_3, r_1) = 0. \tag{12}$$

Equation 9 is solved numerically producing a set of points (r_3, z_3, z_3') which define the shape of the third mirror. These data are fitted by nonlinear least square method[9] to an aspherical function of the form

$$z_3(r_3) = \frac{c\ {r_3}^2}{1 + \sqrt{1 - (1+k)(c\ r_3)^2}} + A_4{r_3}^4 + A_6{r_3}^6 + ... + A_n{r_3}^n. \tag{13}$$

The relation between the points (r_1, z_1) and (r_3, z_3) is given by the Abbe sine condition. From this relation, the ray trace equations and the fitted Eq. 13 , the differential equation for the primary mirror is solved to obtain a set of surface data points (r_1, z_1, z_1') for the primary mirror. The relation between (r_2, z_2) and (r_3, z_3, z_3') is given by the geometry of the system (Fig. 1). Given this relation, the ray trace equations, and third mirror surface equation, the differential equation for the secondary mirror is solved numerically and a set of surface data points (r_2, z_2, z_2') is obtained. As with the third mirror, the surface data for the primary and secondary mirrors are fitted by nonlinear least square method[9] to the aspherical function (Eq. (13)).

The system is reproduced using an optical design and analysis package, such as CODE V,[8] and its performance analyzed.

3. APPLICATION

As a means of comparison, a baseline 3-mirror telescope is designed using conventional optimization techniques to minimize the aforementioned undesirable aberrations. The optical performance of the system is also calculated with CODE V.[8] The baseline system's ray-trace, spot diagram and MTF plot are given in Figs. 2, 3 and 4 respectively.

Taking initial parameters from the baseline system, a new system is designed by employing the techniques outlined in the Theory Section. The new system's ray-trace, spot diagram and MTF are given in Figs. 5, 6 and 7, respectively.

From Fig. 7, one can see the diffraction MTF plot is coincident with the diffraction limited MTF curves. Therefore, the new system has achieved the diffraction-limited MTF performance. The spot diagrams are very small ($10^{-5}\mu m$), as shown in Fig. 6. There is no astigmatism or field curvature for the new system.

4. CONCLUSIONS

While the system designed via the differential equation method has very good performance (at least as good as the optimized system), it should be noted that only parallel incident rays are used in its design–that is, α is set to zero when computing the shape of the system shown in Fig. 5. In the future, we will consider systems where α varies appropriately over the some preset angle. Thus, we hope to show that this method can produce multi-mirror systems that are nearly aberration-free while maintaining a wide field-of-view.

5. REFERENCES

1. F. A. Jenkins and H. E. White, *Fundamentals of Optics*, chap. 3, McGraw-Hill, New York, 1957.

2. H. P. Brueggemann, *Conic Mirrors*, chap. 1, Focal Press, New York, 1968.

3. D.G. Burkhard and D. L. Shealy, "Simplified formula for the illuminance in an optical system," *Appl. Opt.* **20**, pp. 897-909, 1981.

4. D. Korsch, *Reflective Optics*, Academic, New York, 1991.

5. C. Wang and D. L. Shealy, "Multi-mirror anastigmat design," *Proc. Soc.Photo-Opt. Instrum. Eng.*, **2000**, pp. 28-33, 1993.

6. S. H. Chao and D. L. Shealy, "Design of an anastigmat two-mirror microscope," *Proc. Soc. Photo-Opt. Instrum. Eng.*, **2540**, pp. 21-27, 1995.

7. D. Korsch, "Design and optimization technique for a three-mirror telescope," *Appl. Opt.*, **19**, pp. 3640-3645, 1980.

8. CODE V is a registered trademark of Optical Research Associates, 3280 E. Foothill Blvd., Pasedena, CA, 91107.

9. J. A. Meddler and R. Mead, "A simplex method for function minimization," *Computer Journal*, Vol. 7, p. 308, 1965.

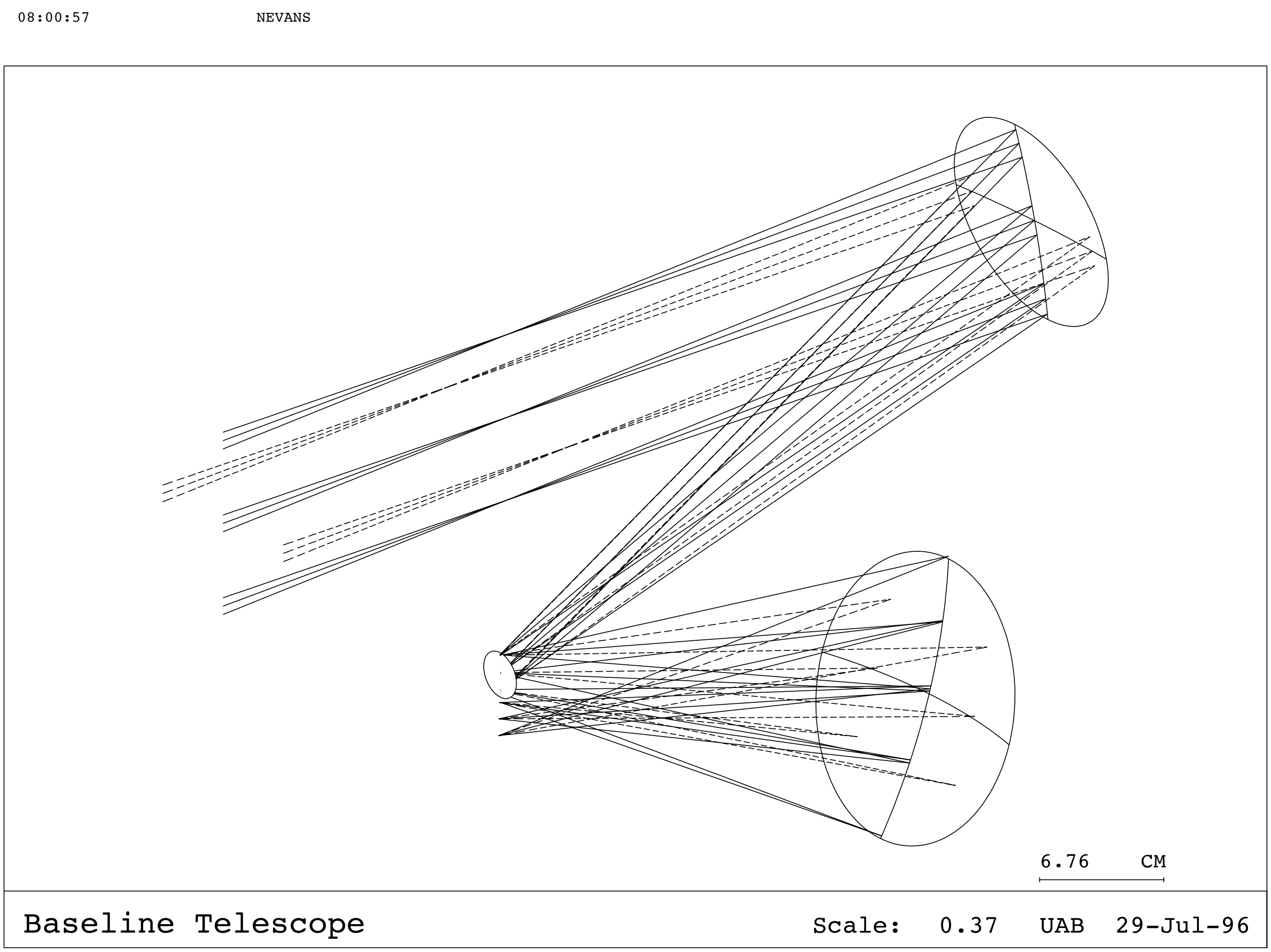

Figure 2: The base-line system.

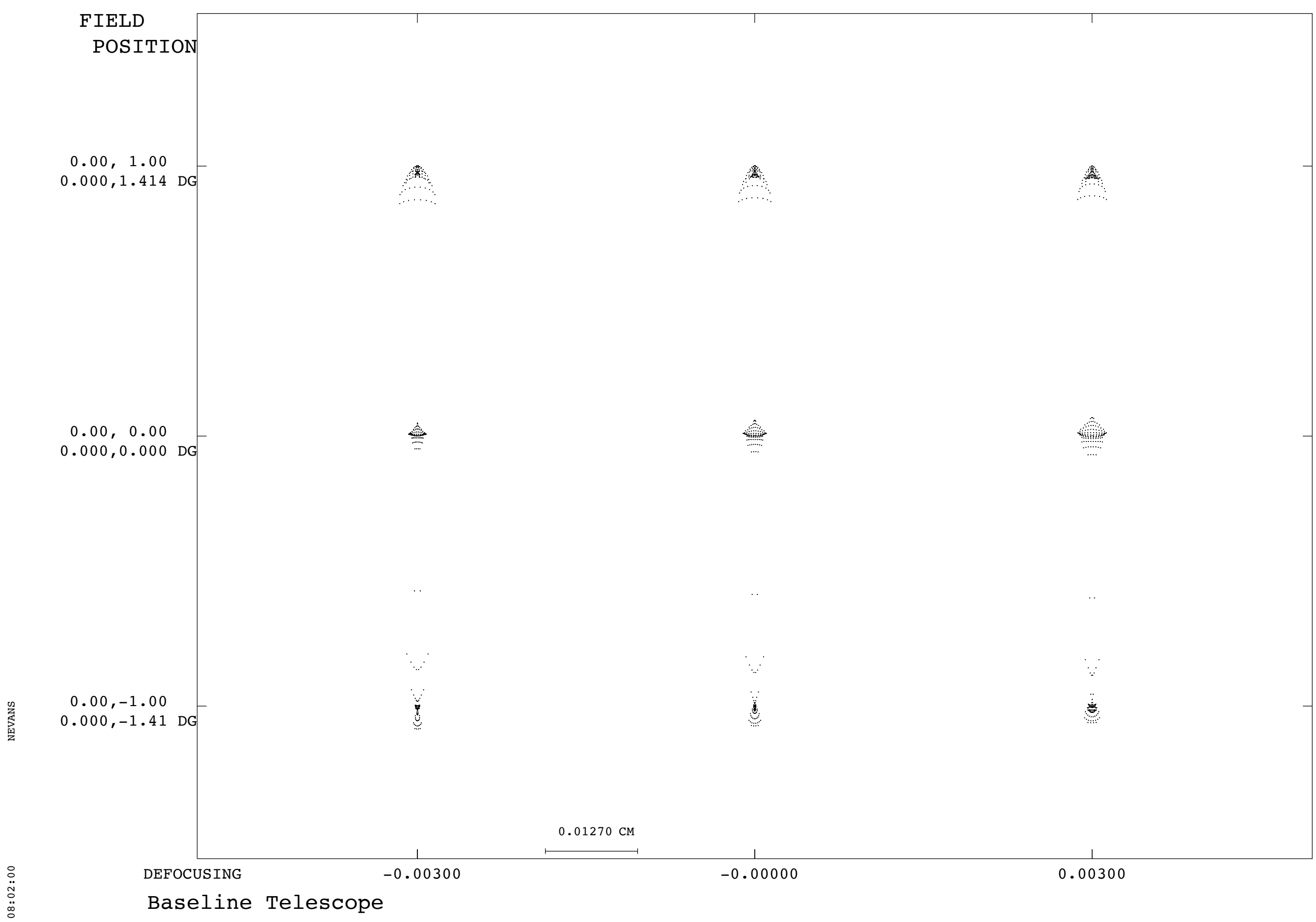

Figure 3: Spot diagram of the base-line system.

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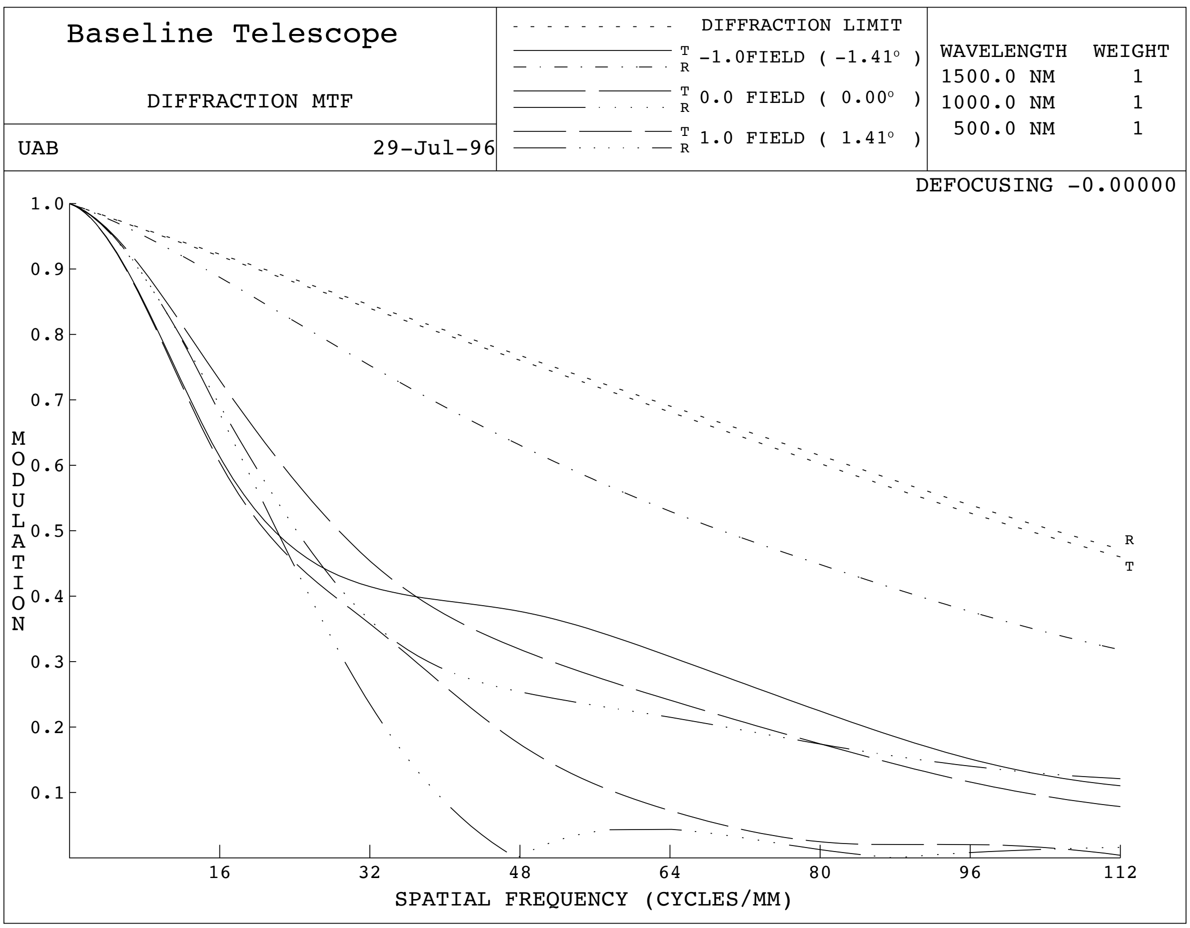

Figure 4: MTF plot of the base-line system.

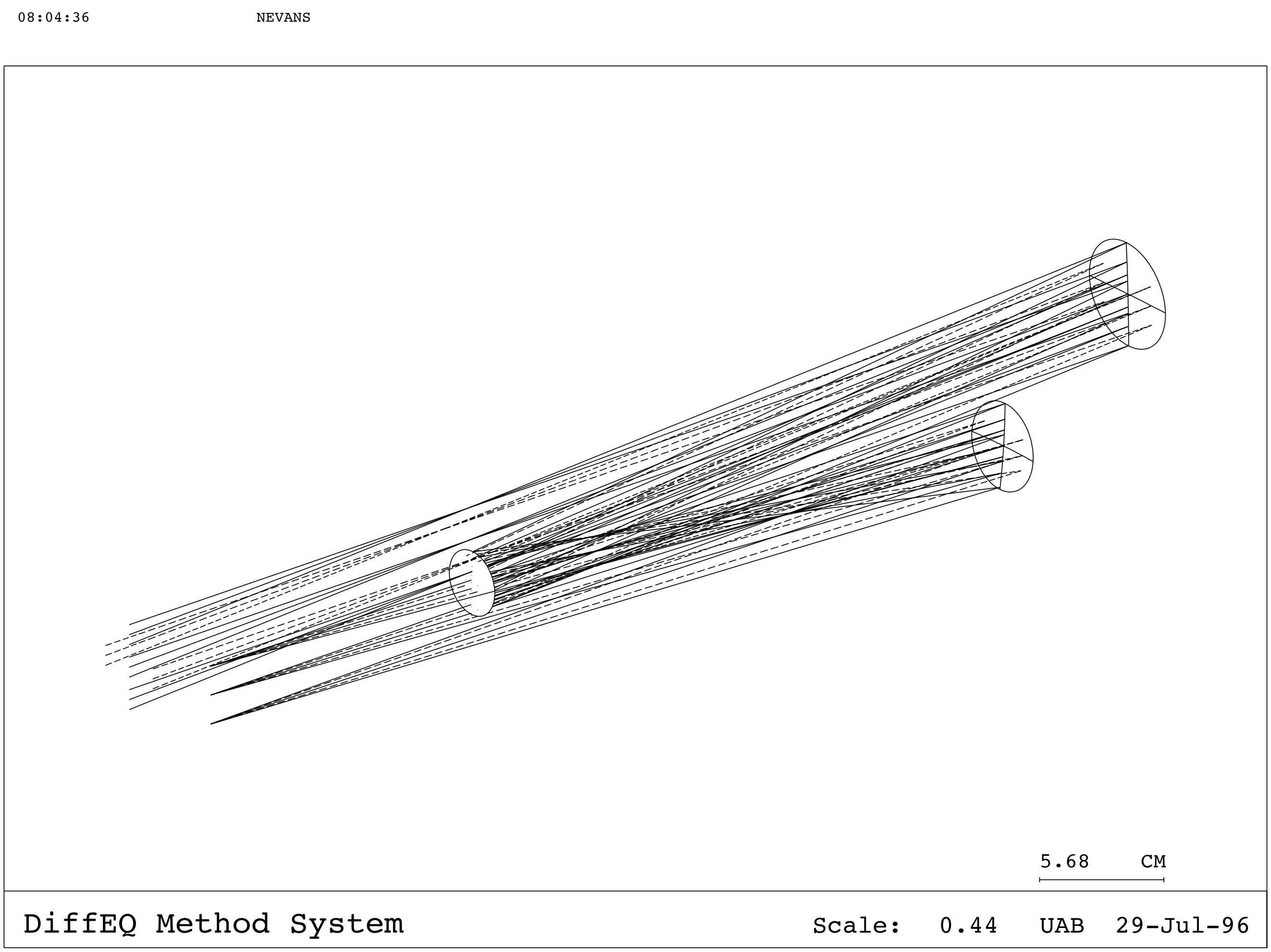

Figure 5: The 3-mirror telescope designed with differential equation method.

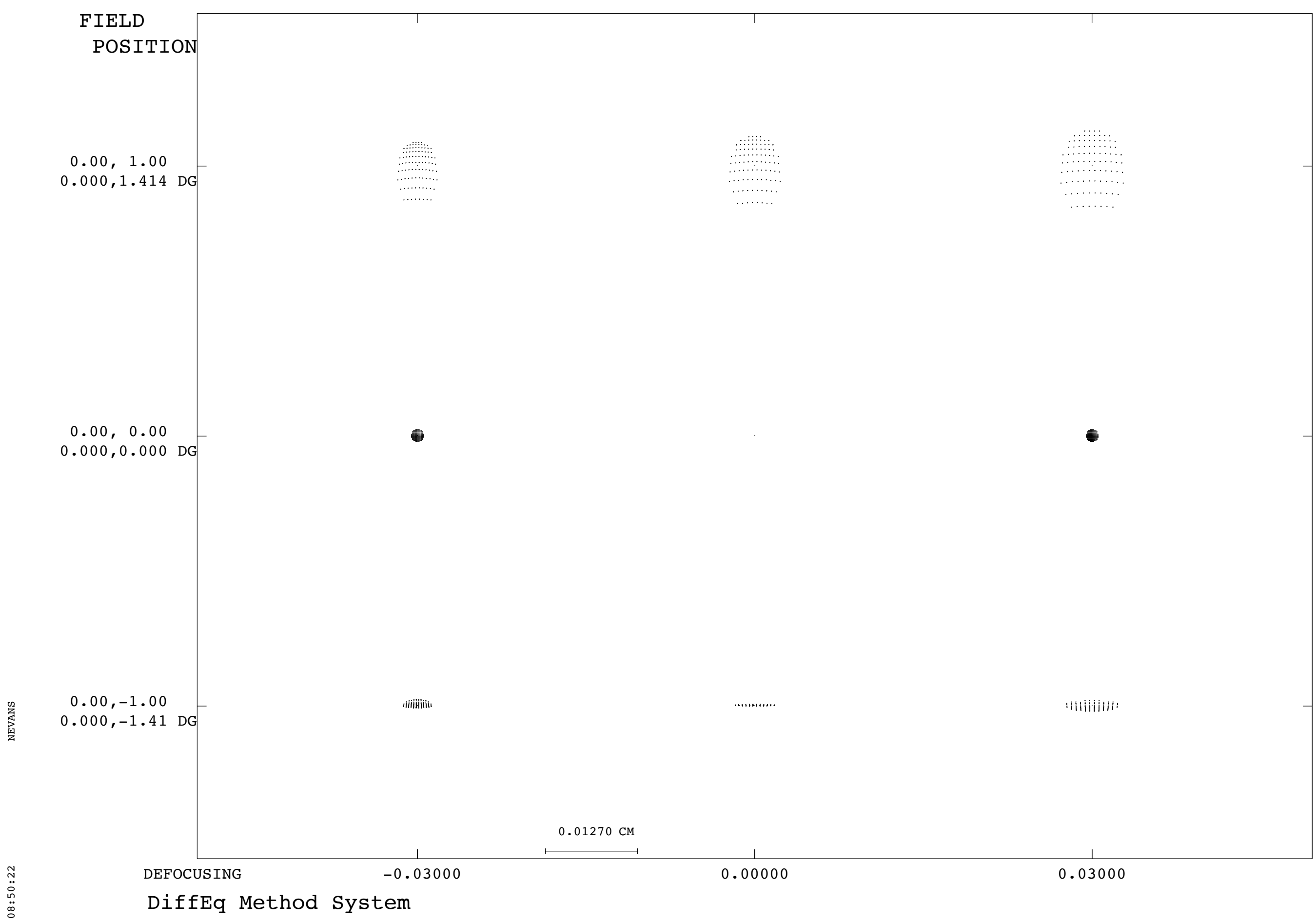

Figure 6: Spot diagram of the differential equation system.

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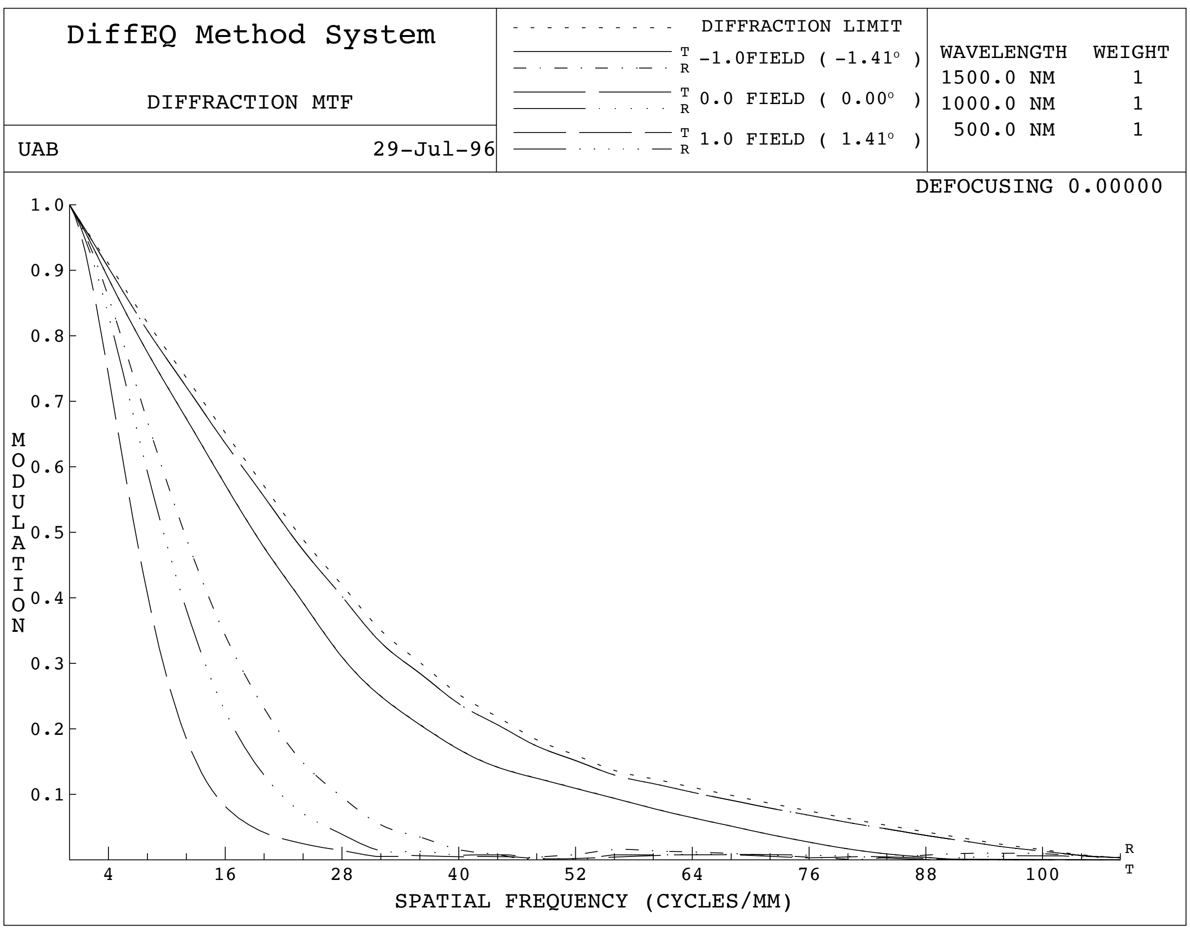

Figure 7: MTF plot of the differential equation system.